The Notion of Epistemological Obstacle in My Works [1995a] & [1995b] -reprinting-

At first, it is a main purpose that [1995*a*] develops categories to describe the process of overcoming the epistemological obstacle of the student. The reason to make categories of description for is as follows. How can we recognize that a student overcomes an epistemological obstacle? I think that a theoretical (or "theory-laden" by words of Hanson) viewpoint is needed for us so that we can understand that a student overcomes an obstacle. The categories of description are prepared for as such our viewpoint.

As a result, we need to distinguish three categories for description of students' overcoming process; "notion", "event", and "conviction"; The "notion" category includes students' ambiguous ideas, images, and mental models. The "event" category means students' concrete experience which "notion" is laden. What is described by the above two categories means the overt behavior of a student. The conviction means the covert and comprehensive value judgment of the student which explains why a student shows such a behavior. In other words, the "conviction" category means a student's attitudes towards mathematical knowledge.

A set of "notion" and "conviction" forms a student's way of knowing. The "notion" also corresponds with some "event". A student doesn't evaluate the "notion" itself directly, but through the "event" as indeed. Hence the "conviction" is also a criteria to evaluate the event. These constitute the following $\mathbb{C}(\mathcal{C}, \mathcal{N}, \mathcal{E})$ model.(*see* ***fig.A***)

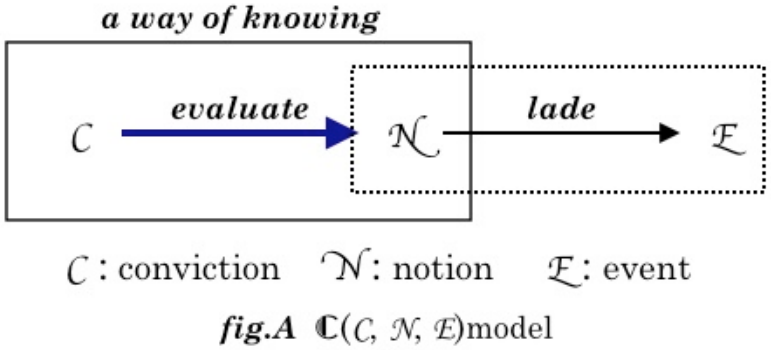

fig.A $\mathbb{C}(\mathcal{C}, \mathcal{N}, \mathcal{E})$model

The process of the student's overcoming epistemological obstacle is explained as follows by this model. If a model is consistent, the student's cognitive activity based on it lasts. Because the consistency of the model is not kept, the learning activity is caused. This means such situation that though a student was able to lade a "notion" ($\mathcal{N}_1$) to a "event" ($\mathcal{E}_1$) in the former activity, which was based on "conviction" (C_1) he/she is going to lade the same way of knowing to the "event" ($\mathcal{E}_2$) that he/she confronts newly, but ends in failure. In other words an existing way of knowing functions as an epistemological obstacle($\mathcal{EO}$).

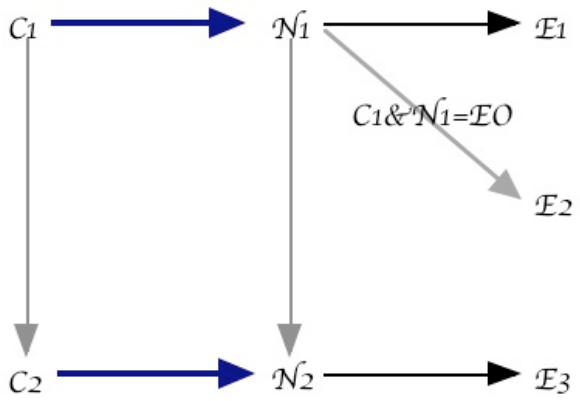

fig.B The emergence of EO

To overcome the epistemological obstacle is not achieved by the revision of the error or the supplement of a necessary matter, if, at least, a way of knowing that a student achieved was satisfied enough in past learning. That is, only to prepare a new "notion"($\mathcal{N}_2$) is not enough. On this account it calls for total reform (or "refonte totale" by words of Bachelard) of the knowing for overcoming epistemological obstacle. That is, it is considered that the overcoming is accomplished by the "notion"(N_2) which is applied to the "event"($\mathcal{E}_3$) confronted newly and the "conviction"(C_2) which evaluates ($\mathcal{N}_2$ & $\mathcal{E}_3$) occurring newly. $\mathcal{E}_2$ - $\mathcal{E}_3$ may be the same object superficially, but they are distinguished as a student looks at from a different viewpoint because his/her way of knowing changes.

When I observe the student's activity with $\mathfrak{C}(C, \mathcal{N}, \mathcal{E})$ model, he/she is not going to overcome an epistemological obstacle anytime even if a teacher

expected it, rather many cases which he/she does not so is seen. When investigating how a student confronts an obstacle, we can identify four different kinds of state of knowing as follows. [1995*b*]

a) *Persistence to the subjective ease*

It leads clear decision making for a student that he/she maintains an existing way of knowing of his/her own which functioned effectively till now at least. Because such a way of knowing is suitable for the past experience of the student. We can call a state of knowing as such a decision making of the student "persistence to the subjective ease".

The state of knowing is characterized such that the student's existing way of knowing does not change and therefore he/she is not going to accept “event” which it does not suit well. (*see* ***fig.C*1**)

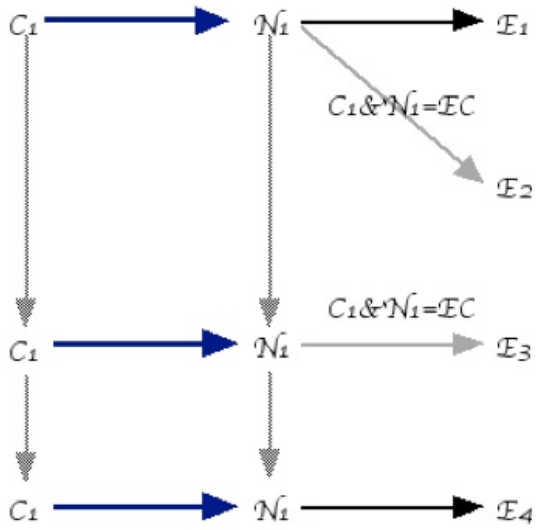

fig.C1 *persistence in subjective facility*

b) *Justification as the social adaptation of “event”*

When a student persists in subjective ease, the relations with another person do not occur. On the other hand, it is seen in the lesson that the student accepts “event” to meet newly with by relations with other(s). We can call such a state of knowing “justification as the social adaptation of “event” ”. The social adaptation to say here is the action of the student who takes it to avoid that it becomes difficult to continue communal living because of the *ill-*

adaptative action. When we run a social life, we perform some kind of social adaptation. This is similar about students in the classroom, and it is necessary. However, students are enough by increasing as information by new knowledge additionally if they merely aim at the social adaptation. (*see* ***fig.C2***) Hence, the state of knowing is characterized by dependence to other(s) of the justification surfaced as words of the students that "*I was taught it by my teacher like that*".

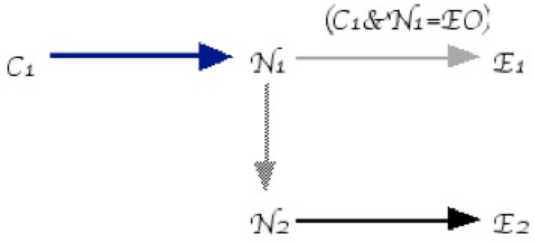

fig.C2 *Justification as the social adaptation*

c)*Becoming aware of an epistemological obstacle*

It is required that a student becomes aware of an epistemological obstacle so that he/she overcomes it. We can call such a state of knowing of the student "Becoming aware of an epistemological obstacle". What is important here is not that a student accepts a new "notion", but that student oneself becomes aware of an epistemological obstacle. Actually, as a result of becoming aware of an obstacle, the student may dismiss a new "notion" or otherwise reserve it. However, we cannot expect the attainment of the overcoming an epistemological obstacle in this state of knowing immediately. It is shown in ***fig.C3***, even if "notion"($\mathcal{N}_3$) which is consistent in "event"($\mathcal{E}_2$) that a student confront with newly was temporarily accepted by him/her, he cannot justify $\mathcal{N}_3$ and therefore he/she may dismiss it since it is evaluated by former "conviction"(C_1). Therefore, this state of knowing is characterized as the activity/thinking of the student is delayed, as a result that he/she becomes aware of his/her own obstacle.

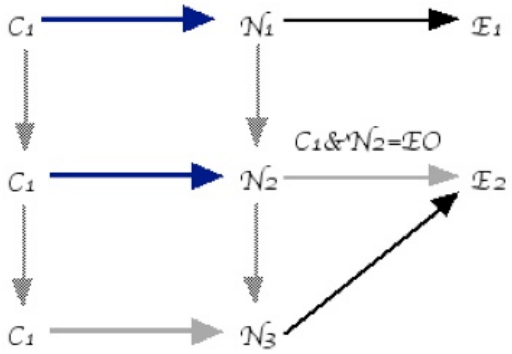

fig.C3 *Becoming aware of an epistemological obstacle*

d)*Overcoming an epistemological obstacle*

When a student is going to overcome an epistemological obstacle, though it is required to become aware of it at first, however why he can overcome an obstacle is that it is by *shifting "conviction"*(C_1 *to* C_2) that he can justify "notion"($\mathcal{N}_3$) which was generated or shown newly and "event"($\mathcal{E}_3$) which "notion"($\mathcal{N}_3$) is laden. Hence, the state of knowing that we call "overcoming an epistemological obstacle" is characterized by ***shifting "conviction"***.

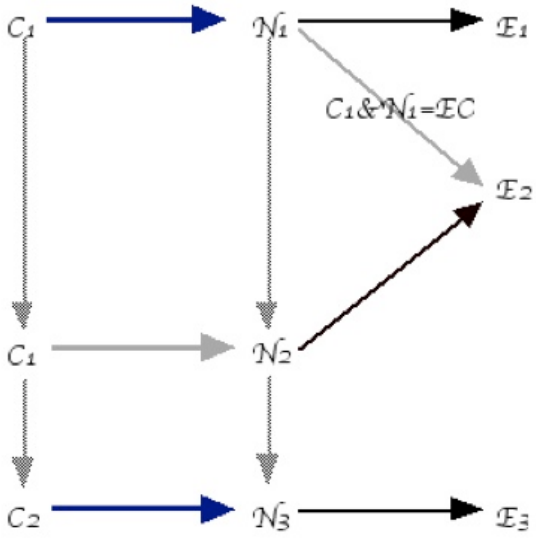

fig.C4 *Overcoming an epistemological obstacle*

Above-mentioned four kinds of state of knowing was all gained from the actual activities of the students by qualitative studies.

The state of knowing that we aim at is "overcoming an epistemological obstacle" obviously. However, the student shows various relations with the obstacle as mentioned above. Four kinds of state of knowing doesn't always have order except that becoming aware of the obstacle is essential for overcoming it, rather they have advantage to each. Nevertheless, if we aim at overcoming the obstacle for the student, it is necessary to explain how it is superior to other relations with the obstacle by laying some kinds of point of view.

1) ***Social context: social nature of the mathematical knowledge and of the construction of the mathematical knowledge***

The epistemological obstacle should be overcome by an individual student. However, we cannot have criteria to judge as a student accomplishes the overcoming of the obstacle if it means quite relative change. Even if a student changed as a result, it may be judged that he/she overcame an obstacle superficially at least, if the obstacle comes not to function as itself. If a student accomplishes such a change, he/she would construct "another mathematics" which is different from the mathematical knowledge that we approve now. (*cf.* Bloor, 1976) That is, it is "mathematics" considered as to deviate from the direction of the development of the knowledge that we aim at in normal education. However, we do not expect the overcoming in such a meaning. Therefore we usually lade certain rationality for "overcoming" tacitly. In other words the overcoming should be done in an individual, but the knowing accomplished by overcoming is not to personal. Because the nature of "another mathematics" that Bloor points out is neither the personal consistency nor the social agreement. That is why we hope that a student learns mathematics as social knowledge as Balacheff(1990) insists. In other words, it is necessary for a student that the solution of the problem is guaranteed in social context. Therefore, *the social context* is the first point of view to describe relation with the obstacle of the student when an observer judges as a student faces an epistemological obstacle, whether or not a student is aware of it. Then is it desirable for us to expect any kind of social context for the mathematics learning of the student?

A student participating in no social context means that he/she does not have any relations with another person and then he/she persists in his/her subjective ease. As mentioned above, the persistence to the subjective ease

has a superior point to arrive at clear decision making for a student. However, when a student meets with another person insisting on a different thought, the student must persuade his thought to another person or do some kind of reaction for the insistence of another person. Therefore, it is impossible that the student ignores the relations with another person, and we also do not expect so. Therefore, there will be seen some kind of social context unless a student persists in his/her subjective ease.

Justification as the social adaptation of "event" means the social context to be concluded among a student with a teacher in particular. This is going to be overt by a student evading a risk with not accepting the taught by a teacher. However, there is not the necessity of the social construction of the mathematical knowledge which is another insistence of Balacheff. As mentioned above, the social adaptation is needed in itself, and if authority is more excellent, the social adaptation based on it is more certain and continues for a long time. (*cf*. Peirce, 1877) However, we do not wish to carry out such authority in the mathematics learning of the student. Rather we expect that a student evolves his/her knowing through judging validity of his/her mathematical knowledge by a confrontation with another person. The phase of such a confrontation occurs in a state of becoming aware of an epistemological obstacle and in a state of overcoming it by assuming the former. Because, in these states, a new "notion" has some kind of context of purpose and meaning, so that the student confronts his/her "conviction" for such a purpose and a meaning. The "notion" is socially demanded based on examining a certain reasoning and its conclusion(s), and the student who is in a state of becoming aware of the obstacle cannot justify such a "notion". In other words, even if the student in such a state accepts the need of the social construction of the mathematical knowledge, he/she cannot accomplish this enough successfully. On the other hand, the state of overcoming epistemological obstacle means that social construction of the mathematical knowledge is achieved.

In summary, we need *social nature of the mathematical knowledge* and *of the construction of the mathematical knowledge* as *social context* in the mathematics learning of the student. The former is not seen in persistence to the subjective ease. It is seen by a way to accept social knowledge with authority in justification as the social adaptation. However, in becoming aware of or overcoming the epistemological obstacle, the student will confront

another person not only for accepting social established knowledge, but also for his/her knowledge to become socially.

2)*Reflection on one's knowing*

It is necessary for a student to be able to become aware of difference between of his/her knowing and of another person so that a student confronts another person. In other words, it is necessary for the student to be able to reflect to his "notion" and "conviction". When a student learns mathematics, it is indispensable for him/her to reflect on his/her knowing so that he/she considers it as not to be fix but to be changeable. Therefore, *reflection on his knowing* is the second point of view to describe relation with the obstacle of the student. Obviously because the phase of the confrontation with another person does not occur in a state of persistence to the subjective ease or of justification as the social adaptation, a student does not reflect on his/her knowing.

What is emphasized in particular by this point of view is the distinction between a state of persistence to the subjective ease and of becoming aware of the obstacle. Let's suppose that a student dismisses a new "event" as a result that he/she becomes aware of an epistemological obstacle. At least, the rejection by a student is not different as a result that he/she persists in subjective ease. However, we distinguish two kinds of state of knowing in a process, not considering both as the same with the result. Then, we can distinguish both definitely by adopting a point of view of reflection.

3) *The "conviction"-shift*

We saw that overcoming epistemological obstacle is accomplished by a student shifting his/her "conviction". It means that overcoming the obstacle is not accomplished only by a student merely correcting his/her old "notion" through reflection of his/her knowing and changing it into a new one. Shifting "conviction" means that conceptual change should be total reform of the knowing not the change of the "notion" only - $\mathfrak{C}(C, \mathcal{N}, \mathcal{E})$ model totally becoming new [1995a] -, and there we see evolution of the knowing of the student. Therefore, *the "conviction"-shift* is the third point of view to describe relation with the obstacle of the student.

What is emphasized in particular by this point of view is to distinguish a state of overcoming epistemological obstacle from a state of becoming aware of the obstacle. In this case also, both advance in comparison with the past so that a student accepts a new "notion" if we see only a result. However, in a state of justification as the social adaptation, his/her "conviction" is maintained because there is not reflection on his/her knowing. On the other hand, overcoming the obstacle means to be able to shift old "conviction" to the new one which justifies a new "notion" so that a student understands social nature of the construction of the mathematical knowledge and then reflects on his/her knowing.

The above-mentioned argument is arranged as the following table. "–" means that the right side is not considered when the left is not seen, because the left becomes the premise of the right. And in a point of view of social context, "△" means that social nature of the mathematical knowledge is seen but social nature of the construction of the mathematical knowledge is not. In the table, it is shown that state of overcoming epistemological obstacle satisfies all points of view. This means that we may not sense that a student follows a process of overcoming epistemological obstacle if his/her learning of mathematics is not interpreted at each point of view. Therefore, three above points of view are suggested as the significance which a student follows a process of overcoming epistemological obstacle.

Relation of a student with the obstacle

State of knowing	***Social context***	***Reflection***	***"conviction"-shift***
Persistence	×	–	–
Justification	△	×	–
Becoming aware	○	○	–
Overcoming	○	○	○

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- Mizoguchi, T. (1995b). A study of the significance of overcoming epistemological obstacle to the learning of mathematics: Focus on the learner's way of concerning with epistemological obstacle. Bulletin of Institute of Education, University of Tsukuba, 20(1), 37-52. (in Japanese)